Fresh Product Supply Chain Decisions Involving Transportation Discount

GUO, Chenkai

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Thesis/Assessment Committee

Professor Cheng, Chun-Hung (Chair)
Professor Cai, Xiaoqiang (Thesis Supervisor)
Professor Leung, May-Yee, Janny (Committee Member)
Professor George Vairaktarakis (External Examiner)

摘要

本文研究的問題是當生產商要運輸一批易腐爛產品給銷售商。這些易腐爛產品要經過比較長的運輸路程才可以運輸到銷售商。銷售商面臨產品在運輸途中腐爛的風險。市場上的需求跟產品的新鮮度和銷售商的價格相關。銷售商需要通過生產商的批發價格，產品到達市場時的新鮮度和隨機的市場需求來決定訂貨量和銷售價格。另外，生產商要根據銷售商的訂貨量決定自己的批發價。很明顯，在集中型供應鏈中(centralized supply chain)，供應鏈的利潤比分散型供應鏈中(decentralized supply chain)高。我們提出一系列的運輸降價的激勵方式(transportation discount incentive scheme)使得生產商和銷售商之間互相合作。這種激勵方式主要集中在生產商對銷售商運輸費用的補償。

同時，本文還研究了在生產商運輸一批易腐爛產品給銷售商的過程中，由第三方物流公司承當運輸任務。第三方物流公司會根據生產商的批發價格訂立一個確保到達時間(guaranteed arrival time)給銷售商。如果第三方物流公司不能在規定的時間內把貨物送給銷售商，那麼他就要和銷售商一起承擔一定的運輸損失。我們對這樣的問題進行了建模，同時對集中型供應鏈和分散型供應鏈的最優化問題進行了研究。最終我們提出了一個生產商和銷售商之間互相合作的機制。機制分為兩個部分：首先，生產商對賣給銷售商的每一件產品都根據產品的到達時間和銷售商的訂貨量給與一定的降價；其次，第三方物流公司根據銷售商的訂貨量和產品的到達時間對因逾期而需要賠付給銷售商的比例進行規定。在這兩種機制的共同作用下，整個供應鏈到達了互相協作的效果。分散型供應鏈的訂貨量與集中型供應鏈的訂貨量相同，同時供應鏈中的參與者都可以到達比不協作時多的利潤。我

們還對模型進行了數值計算，從中可以看出一些供應鏈中重要的管理理念。

關鍵字：供應鏈協作，第三方物流，遠距離運輸，易腐爛產品，銷售價格機制，賠付機制。

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This paper considers a supply chain in which the producer sells a large batch of fresh products to the distributor. The products face a long distance transportation before they arrive at the distributor side. The distributor faces the risk that the product he has procured may decay and deteriorate during transportation. The market demand for his product depends on its level of freshness and his selling price. The distributor has to determine his order quantity and selling price, taking into account the wholesale price of the producer, the possible loss of the product in the long distance transportation, the product's level of freshness when it reaches the market, and the uncertain market demand for his product. On the other hand, the producer has to determine his wholesale price based on the effect of the order quantity from the distributor. Apparently, the expected centralized supply chain profit is larger than that of the decentralized system. We introduce a series of the Transportation Discount Incentive (TDI) schemes which facilitate the coordination of the two parties. The scheme will focus on the transportation compensation offered by the producer to the distributor.

Meanwhile, this paper also considers a supply chain in which

the producer sells a large batch of fresh products to the distributor. And a third-party logistics (3PL) provider will help the distributor deliver the product. The 3PL provider will propose a guaranteed arrival time to the distributor based on the wholesale price. If the 3PL provider can not deliver the product on time, he has to share a portion of the transportation risk with the distributor. We develop a model to formulate this problem and derive each party's optimal decisions in both uncoordinated and coordinated situations. We also propose a coordination mechanism which comprises two parts: (i) the producer offers a wholesale pricing policy as a function of the actual arrival time and the distributor's order quantity; and (ii) the 3PL provider offers a penalty factor policy as a function of the distributor's order quantity and the actual arrival time. We show that this coordination mechanism can induce the distributor to order up to the quantity required to maximize the total benefit of the centralized system, and both parties will be better off through this mechanism. Computational studies is also conducted, which reveal some important managerial insights in the supply chain management area.

Keywords: Supply Chain Coordination, Third-party Logistics, Long Distance Transportation, Perishable Products, Wholesale Pricing Policy, Penalty Factor Policy.

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This work is dedicated to my family.

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Chapter 1

Introduction and Literature Review

The production of fresh products, such as live seafood, fresh fruit, fresh vegetables, etc., is highly dependent on the geographical locations. Especially in Hong Kong, most of the fruit and vegetables are imported from America or Southeast Asia and some high quality seafood, for example, lobster, fish fin and crab, is coming from Australia. Similar cases are prevailing in China recently. Some of the largest third-party logistics services provider built up their own distribution centers in the major cities of China providing fresh product transportation services.

There are a large variety of papers studying the importance of long distance transportation. For instance, paper [8] highlights that more than 485,000 truckloads of fresh fruit and vegetables leave California every year. Revenues and costs for shipping these loads were determined for shipments going to five cities: Atlanta, Chicago, Dallas, Denver and New York City. Obviously, the long distance transportation faces a high risk. Due to the perishability and time passed, the products will have a certain degree of decay or deterioration. As indicated by [9], "All fresh products continue to deteriorate with time, even under optimum handling and transport conditions. Post harvest and transport times should be kept as short as possible, especially under less than optimum conditions, to limit deterioration and extend marketable life of products." There are a lot of factors that influence the long distance transportation, for example, weather conditions, equipment breakdowns, transportation jam, etc. When such a case happen, the product will have a significant decay and deterioration during the transportation process.

The main purpose of this thesis is to study on those products that have a very short lifespan. We build up three transportation discount incentive (TDI) schemes based on [4]'s model which has developed to capture the key features and concerns of supply chain management of fresh products involving long distance transportation. We will consider two scenarios: (1) The decentralized supply chain system, where the distributor and the producer make decisions to optimize their respective objectives; and (2) The centralized system, where the two parties make decisions to optimize a joint objective. Under the TDI schemes, the two parties in the decentralized supply chain could be motivated to make coordinated decisions so that the joint objective is optimized and all of them will be better off.

Meanwhile, We also develop a model including a Third-Party Logistics (3PL) provider who helps the producer arrange the shipping issue. Currently, more and more customers complain about the delay of the product arrival time. Especially for perishable products, the longer the time, the lower the quality is. For simplicity, we assume the product perishability directly relates to the transportation time, which may be affected by the scheduling of 3PL provider. The product decay and deterioration will be captured using similar form to [4] and [5]. And the transportation cost will be paid by the distributor. The 3PL provider will offer a guaranteed arrival time to the distributor. If the product arrives late, the 3PL provider will pay some penalty cost to the distributor to eliminate the transportation risk faced by the distributor. The main problem that we investigate is the supply chain involving three parties, the producer, the distributor and the 3PL provider. The whole supply chain is dealing with the long distance transportation of fresh products. We will study all the optimal decisions for the three parties involved in the supply chain system. Moreover, we also explore our research in the supply chain coordination area. That is, the producer will propose a wholesale pricing policy to the distributor for all the unit purchased. And the distributor will try to sign a contract with the 3PL provider, which indicates the penalty cost suffered by the 3PL provider.

This thesis is closely related to the supply chain management of perishable products. An early work describing the perishable inventory problem is done by [20] where the products deteriorating at the end of the storage periods were considered. Since then, more and more scholars starts their attention to the research on the perishability of the product. [10] provides a comprehensive survey of the research results published before the 1980s, where the perishable products lifetime were classified into two groups: fixed and random. Recent studies on the deteriorating product models can be found in [14] and [7]'s reviews, where the relevant literature published in 1980s and 1990s was reviewed respectively. The major concern of these literature is either focus on the quantity loss or the value drop. While [15] considers both the quantity loss and value drop. Referring to paper [4], it describes a perishable product supply chain which faces long distance transportation. The demand of the model is random and price sensitive. And the quantity loss will affect the effective supply and the value drop will affect the corresponding demand. [4] points out that the decentralized supply chain losses significant expected profit compared with the centralized system because of the independent decision making. Then, [4] proposes an incentive scheme which consists of a wholesale price discount contract and a buy back contract so as to facilitate the coordination issues.

The supply chain coordination has been a hot research subject for the last few decades. One line of the literature is usually achieved through contracts between the upstream producers and downstream distributors, to increase the total supply chain profit so that the decentralized supply chain acts closely to the case of the centralized supply chain. Various models of supply chain contracts have been developed in the literature. Price discount is often suggested as an incentive to facilitate coordination. (see [13],[18] and [19]). Other incentive schemes include quantity commitment (see [6]), buy back or return policies (see [12]), revenue sharing (see [2]), sales rebate or markdown allowance (see [17]) and paper [3] provides an excellent survey on supply chain coordination with contracts.

As was mentioned in the introduction, this paper is a continuum of paper [4] and [5]. In paper [4], the supply chain coordination is achieved through a wholesale price discount contract and a buy back contract. In our thesis, we try to explore to use one contract which only relates to the transportation time and the shipping quantity to induce the coordination strategy. Paper [5] develops a supply chain model involving 3PL provider. The difference of our thesis is that we introduces a guaranteed arrival time as the 3PL provider's decision, while paper [5] sets the 3PL provider's transportation cost as the decision variable. Meanwhile, our thesis continue to use the FOB business model like paper [4], which the transportation cost is paid by the distributor, while paper [5]'s transportation cost is paid by the producer.

Section 2, we discuss the details of our basic model and some basic results done by paper [4]. Following section 2, we will introduce different TDI schemes in section 3. And the 3PL logistics provider formulation, notation and assumptions will be introduced in section 4. Section 5 conducts some computational studies for the comparison of TDI schemes and the schemes with the 3PL provider. Finally, the conclusion will be stated in section 6.

□ **End of chapter.**

Chapter 2

Problem Description and Preliminary Results

We investigate the following model. A producer sells the perishable product to a distributor. Decay and deterioration may take place during transportation, which reduce the quantity and the quality of the product. The product is highly perishable and the salvage value will be zero if it can not be sold within the lifespan. The time period of the product lifespan is defined as a single time period. Meanwhile, the product faces a long distance transportation whose cost is paid by the distributor. So the transaction between the producer and distributor is based on free-on-board (FOB). The distributor has to determine two decisions: the order quantity from the producer and the selling price to the end customer in the market. The producer has to determine the wholesale price based on the order quantity from the distributor.

The product is fully fresh when it is loaded to the cargo ship. There exists a perishable duration τ which states the product is fresh within the τ time period, where $\tau \geq 0$. After that, the product starts to decay at a significant rate. We define $\theta(t)$ as the freshness level of the product. $\theta(t) = 1$ when $t < \tau$ and $0 \leq \theta(t) < 1$ otherwise. And a function m(t) is defined over

[0,1] as the index on the marketable quantity of the product at time t. t is random in the model.

Consider a product with uncertain and price-sensitive demand during a single selling period or season. Assume that demand for the product, denoted by D, has the following multiplicative functional form:

$$D(p) = y(p) \cdot \epsilon \tag{2.1}$$

where $y(p)$ is a deterministic and decreasing function of the product's selling price p, and ϵ is a random factor with CDF $F(\cdot)$, PDF $f(\cdot)$. Then let $y(p)$ take the form of

$$y(p) = ap^{-b}\theta(t) \qquad \text{where } a > 0,\, b > 1. \tag{2.2}$$

where b is a price-elastic index and a is a fixed constant. We focus on price-elastic products by assuming $b > 1$. (If $b \leq 1$, then we can show that the optimal price under consideration goes to infinity.) In addition, we also assume C_T is the transportation cost per unit. w is the wholesale price per unit, c is the production cost per unit for the producer, q is the order quantity of the distributor and p, q, w are decision variables.

2.1 Traditional Decentralized Supply Chain

In the decentralized supply chain, the producer needs to determine the wholesale price and the distributor needs to determine the order quantity and the selling price to the customer. The producer and distributor will maximize their expected profits independently. The distributor's expected profit is:

$$\pi_d(p, q_d|t) = pE_\epsilon[min(q_d m(t), D(p,t))] - wq_d - C_T q_d \tag{2.3}$$

Where q_d is the order quantity and p is the selling price in the decentralized supply chain. In the situation where there is no coordination between the producer and the distributor, the distributor is to determine p and q_d by maximizing his expected profit $\pi_d(p, q_d)$, where

$$\pi_d(p, q_d) = E_t\{\pi_d(p, q_d|t)\} \tag{2.4}$$

Following [11], this paper uses the stock factor z which defines as $q_d m(t)/[ap^{-b}\theta(t)]$. By substituting $p = (za\theta(t)/q_d m(t))^{1/b}$ into equation 2.3, the distributor's objective function can be rewritten as:

$$\pi_d(z|q_d, t) = (\frac{az\theta(t)}{q_d m(t)})^{1/b} E_\epsilon\{min\{\frac{q_d m(t)}{z}\epsilon, q_d m(t)\}\} - (w + C_T)q_d \tag{2.5}$$

The paper [4] states the following lemma for the optimal stock factor.

Lemma 2.1*[Cai et al., 2008]* *The optimal stocking factor z that maximizes $\pi_d(z|q_d, t)$ is determined by*

$$\frac{b-1}{b}[z - \Lambda(z)] = z[1 - F(z)] \tag{2.6}$$

where

$$\Lambda(z) = \int_0^z (z - x)f(x)dx$$

Moreover, if ϵ has a generalized increasing failure rate, and $\lim_{x\to\infty} x[1 - F(x)] = 0$, then equation 2.6 has a unique solution z_0.

From equation 2.6, we see that the optimal stock factor is dependent on the price-elasticity index b and ϵ which shows the market fluctuation, but independent of other parameters. Then the optimal selling price is obtained by:

$$p^*(q_d, t) = (\frac{az_0\theta(t)}{q_d m(t)})^{1/b} \tag{2.7}$$

By substituting $p^*(q_d, t)$ of into 2.3, the distributor's expected profit given time period t is:

$$\begin{aligned} \pi_d(q_d|t) &= \pi_d(p^*(q_d,t)|q_d,t) && (2.8) \\ &= \frac{b}{b-1}[a\theta(t)]^{1/b} z_0^{1/b} [q_d m(t)]^{1-1/b}[1-F(z_0)] \\ &\quad -(w+C_T)q_d && (2.9) \end{aligned}$$

The distributor is to maximize his expected profit $\pi_d(q_d)$:

$$\pi_d(q_d) = E_t\{\pi_d(q_d|t)\} \qquad (2.10)$$

Paper [4] has already got the following optimal solutions for the producer and distributor. The distributor's optimal order quantity is:

$$q_d^* = az_0[\frac{1-F(z_0)}{C_T+w}K_0]^b$$

Where $K_0 = E_t\{\theta(t)^{1/b} m(t)^{1-1/b}\}$. The corresponding optimal expected profit is:

$$\pi_d^* = \frac{(w+C_T)az_0}{b-1}[\frac{1-F(z_0)}{w+C_T}K_0]^b \qquad (2.11)$$

For the producer, the expected profit is:

$$\begin{aligned} \pi_m &= (w-c)q_d^* \\ &= (w-c)az_0[\frac{1-F(z_0)}{C_T+w}K_0]^b \end{aligned}$$

And the optimal wholesale price is:

$$w^* = \frac{bc+C_T}{b-1}$$

We define the total expected supply chain profit as the summation of the producer's and distributor's profit. After applying w^*, q_d^* and p^*, we can get the optimal expected profits for the producer π_d^* and distributor π_m^* respectively.

$$\begin{aligned} \pi_m^* &= \frac{C_T + c}{b-1} a z_0 [\frac{[1 - F(z_0)](b-1)}{b(C_T + c)} K_0]^b && (2.12) \\ \pi_d^* &= \frac{a z_0 b (C + C_T)}{(b-1)^2} [\frac{[1 - F(z_0)](b-1)}{b(C_T + c)} K_0]^b && (2.13) \end{aligned}$$

2.2 Centralized Supply Chain

In the centralized supply chain, the producer and retailer are treated as a joint party. And the expected profit function of the centralized supply chain is:

$$\begin{aligned} \pi_c(q_c) &= E_t\{\pi_c(q_c|t)\} \\ &= E_t[p_c^*(q_c, t) min(q_c m(t), D(p_c^*(q_c, t), t))] - (C_T + c) q_c \end{aligned}$$

Where q_c is the order quantity and $p_c^*(q_c, t)$ is the selling price. Note that the optimal selling price depends on the time t and the order quantity q_c and based on definition of the stock factor, $p_c^*(q_c, t) = (za\theta(t)/qm(t))^{1/b}$. Paper [4] has figured out the optimal order quantity and expected profit respectively:

$$q_c^* = a z_0 [\frac{1 - F(z_0)}{c + C_T} K_0]^b \tag{2.14}$$

$$\pi_c = \frac{1}{b-1} a z_0 (c + C_T) [\frac{1 - F(z_0)}{c + C_T} K_0]^b \tag{2.15}$$

Until now, we have already figured out the optimal decisions for both the producer and distributor in centralized and decentralized supply chain system respectively. As mentioned before, we define the total decentralized supply chain profit as the summation of the producer's and the distributor's profits. In the centralized supply chain, the total profit is observed by equation 2.15. For the decentralized supply chain, the total expected profit π^*_{total} is as follows:

$$\begin{aligned}
\pi^*_{total} &= \pi^*_m + \pi^*_d \\
&= \frac{b}{(b-1)^2}(c+C_T)az_0[\frac{[1-F(z_0)](b-1)}{b(C_T+c)}K_0]^b \\
&= \pi_c * (\frac{b-1}{b})^{b-1} \\
&< \pi_c
\end{aligned}$$

We can see that the decentralized supply chain can not reach the centralized supply chain profit since the producer and distributor make their decisions independently instead of joint decision making in the centralized system.

□ **End of chapter.**

Chapter 3

Transportation Discount Incentive Scheme

This chapter is to introduce three Transportation Discount Incentive (TDI) Schemes to encourage the decentralized supply chain distributor's order quantity up to the centralized supply chain order amount. Then the whole supply chain gets coordinated.

3.1 TDI Scheme I

The first scheme is dealing with the transportation cost. Originally, if the distributor orders q amount of the products, the transportation fee that he should pay will be $C_T * q$. Now we let the producer compensate part of the transportation cost paid by the distributor. The detailed scheme is like follows.

$$\widetilde{C_T}(\alpha, q) = \begin{cases} C_T - \alpha q & \text{if } q \leq \bar{q} \\ C_T - \alpha \bar{q} & \text{if } q > \bar{q} \end{cases} \tag{3.1}$$

q is the decision variable in this scheme, we assume that there exist a $\bar{q}$, which defines the cutting point of the scheme. When $q \leq \bar{q}$, the distributor will get α amount of compensation per unit ordered. Otherwise, the transportation cost will be fixed to $(C_T - \alpha\bar{q})$ per unit.

From paper [4], we have already figured out the optimal selling price depends on the time t and the order quantity q and based on definition of the stock factor, $p_s^*(q_s, t) = (za\theta(t)/qm(t))^{1/b}$. $p_s^*(q_s, t)$ and q_s are the selling price and order quantity under the TDI scheme I.

3.1.1 Optimal Order Quantity for $q \leq \bar{q}$

Let's focus on $q \leq \bar{q}$ case first. The retailer's expected profit under the transportation scheme $\pi_d^s(q_s)$ should be equal to:

$$E_t\{\pi_d^s(p_s^*(q_s, t)|q, t)\}$$

Then

$$\begin{aligned}
\pi_d^s(q_s) &= E_t\{\pi_d^s(p_s^*(q_s, t)|q, t)\} \\
&= E_t[p_s^*(q_s, t)min(q_s m(t), D(p_s^*(q_s, t), t)] - w^* q_s \\
&\quad -(C_T - \alpha q_s)q_s \\
&= \frac{b}{b-1} a^{1/b} z_0^{1/b} K_0 q_s^{1-1/b}[1 - F(z_0)] - (C_T - \alpha q_s + w^*)q_s
\end{aligned}$$

Lemma 3.1 *Let $\gamma(q_s)$ be the function of the first derivative of function $\pi_d^s(q_s)$. $\gamma(q_s)$ is strictly convex.*

Proof The first derivative of function $\pi_d^s(q_s)$ should be

$$\begin{aligned}
\gamma(q_s) &= \frac{\partial \pi_d^s(q_s)}{\partial q_s} && (3.2) \\
&= a^{1/b} z_0^{1/b} K_0 [1 - F(z_0)] q_s^{-1/b} - w^* - C_T + 2\alpha q_s && (3.3)
\end{aligned}$$

If the function $\gamma(q_s)$ is strictly convex, it must satisfy the first and second order conditions.

$$\lambda(q_s) = \frac{\partial \gamma(q_s)}{\partial q_s} = -\frac{1}{b} a^{1/b} z_0^{1/b} K_0 [1 - F(z_0)] q_s^{-1/b-1} + 2\alpha \quad (3.4)$$

Where $\lambda(q_s)$ is the first order condition function of $\gamma(q_s)$.

$$\frac{\partial\lambda(q_s)}{\partial q_s} = \frac{\partial^2\gamma(q_s)}{\partial q_s^2} \quad (3.5)$$

$$= \frac{1}{b} * \frac{b+1}{b} a^{1/b} z_0^{1/b} K_0[1-F(z_0)] q_s^{-1/b-2} > 0 \quad (3.6)$$

we can see that since $\frac{\partial\lambda(q_s)}{\partial q_s} > 0$, the original function $\lambda(q)$ is a monotonously increasing function. Meanwhile, when $q_s \to +\infty$, $\lim \gamma(q_s) > 0$ and when $q_s \to 0$, $\lim \gamma(q_s) > 0$. And based on equation 3.6, we draw the conclusion that $\gamma(q)$ is a strictly convex function which completes the proof.

□

When the first order condition of $\gamma(q_s)$ equals to zero, the corresponding minimum point q^0 is observed as follows.

$$q^0 = [\frac{a^{1/b} z_0^{1/b}[1-F(z_0)]K_0}{2\alpha b}]^{\frac{b}{b+1}}$$

We have already known that $\gamma(q_s)$ is a convex function. Now we discuss the possible situations for $\pi_d^s(q_s)$. There are two possible situations for function $\gamma(q_s)$. One is $\gamma(q^0) \geq 0$, the other is $\gamma(q^0) < 0$.

When $\gamma(q^0) \geq 0$, we see that all the $\gamma(q_s)$ values are nonnegative, which illustrates that $\pi_d^s(q_s)$ will be an increasing function with respect to q_s. Then the optimal order quantity q_s^* will be obtained when $q_s^* = \bar{q}$.

When $\gamma(q^0) < 0$, figure 3.1 illustrates the possible function of $\gamma(q_s)$ and the corresponding $\pi_d^s(q_s)$. There exists a $q_1(\alpha)$ and $q_2(\alpha)$, which divided $\pi_d^s(q_s)$ into three parts. When $q_1(\alpha) \geq \bar{q}$, the optimal order quantity should be $\bar{q}$. While $q_1(\alpha) < \bar{q}$ and $\pi_d^s(\bar{q}) \leq \pi_d^s(q_1)$, $q_s^* = q_1(\alpha)$. Otherwise, $q_s^* = \bar{q}$.

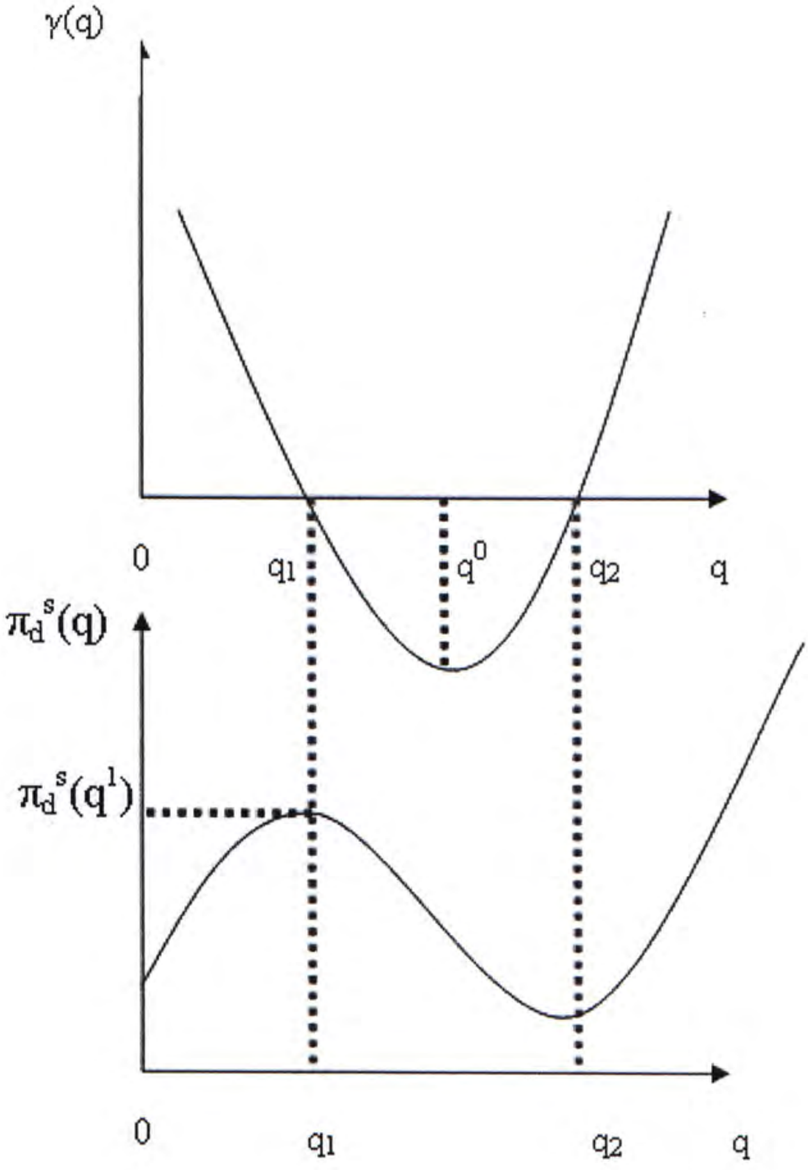

Figure 3.1: $\gamma(q)$ with two intersection points with x-axis

Now we summarized the results for $q \leq \bar{q}$. When $\gamma(q^0) \geq 0$, $q_s^* = \bar{q}$. When $\gamma(q^0) < 0$,

$$q_s^* = \begin{cases} \bar{q} & \text{if } q_1(\alpha) \geq \bar{q} \\ q_1(\alpha) & \text{if } q_1(\alpha) < \bar{q} \text{ and } \pi_d^s(\bar{q}) \leq \pi_d^s(q_1) \\ \bar{q} & \text{if } q_1(\alpha) < \bar{q} \text{ and } \pi_d^s(\bar{q}) > \pi_d^s(q_1) \end{cases} \tag{3.7}$$

Lemma 3.2 *The optimal $q_1(\alpha)$ will exist and unique when the following constraint achieves.*

$$\frac{b}{b-1}(c + C_T) \;>\; (2\alpha a b z_0)^{\frac{1}{b+1}} K_0^{-\frac{1}{b+1}} [1 - F(z_0)]^{\frac{b}{b+1}} [1 + \frac{1}{b} K_0]$$

Proof The necessary and sufficient condition for the uniqueness of optimal order quantity q_1 is $\gamma(q^0) < 0$.

$$\begin{aligned} \gamma(q^0) &= a^{1/b} z_0^{1/b} [1 - F(z_0)][\frac{a^{1/b} z_0^{1/b}[1 - F(z_0)]K_0}{2\alpha b}]^{\frac{-1}{b+1}} - w^* \\ &- C_T + 2\alpha[[\frac{a^{1/b} z_0^{1/b}[1 - F(z_0)]K_0}{2\alpha b}]^{\frac{b}{b+1}}] \\ &= K_0^{-\frac{1}{b+1}} a^{\frac{1}{b+1}} z_0^{\frac{1}{b+1}} [1 - F(z_0)]^{\frac{b}{b+1}} (2\alpha b)^{\frac{1}{b+1}} \\ &+ 2\alpha[\frac{a^{1/b} z_0^{1/b}[1 - F(z_0)]K_0}{2\alpha b}]^{\frac{b}{b+1}} - w^* - C_T \\ &= K_0^{-\frac{1}{b+1}} (2\alpha a b z_0)^{\frac{1}{b+1}} [1 - F(z_0)]^{\frac{b}{b+1}} [1 + \frac{1}{b} K_0] \\ &\quad - \frac{b}{b-1}(c + C_T) < 0 \end{aligned}$$

This completes the proof.

$\square$

3.1.2 Optimal Order Quantity for $q > \bar{q}$

If the order quantity exceeds the cutting point $\bar{q}$, the transportation cost per unit $\widetilde{C_T}$ to the distributor will be fixed at $(C_T - \alpha\bar{q})$. The distributor's objective function will change to:

$$\pi_d^s(q_s) = E_t[p_s^* min(q_s m(t), D(p_s^*, t)] - w^* q_s - \widetilde{C_T} q_s \qquad (3.8)$$

Lemma 3.3 *when $q > \bar{q}$, the optimal order quantity in such situation $q_s^{**}(\alpha)$ will be:*

$$q_s^{**}(\alpha) = az_0[\frac{1 - F(z_0)}{C_T - \alpha\bar{q} + w^*} K_0]^b \qquad (3.9)$$

And the corresponding expected profit is:

$$\pi_d^{s**} = az_0 \frac{w^* + C_T - \alpha\bar{q}}{b-1} [\frac{1 - F(z_0)}{w^* + C_T - \alpha\bar{q}} K_0]^b \qquad (3.10)$$

Proof We know that the optimal order quantity should satisfy the first and second order condition of equation 3.8.

$$\begin{aligned} \frac{\partial \pi_d^s(q_s)}{\partial q_s} &= a^{1/b} z_0^{1/b} K_0[1 - F(z_0)] q_s^{-1/b} - (\widetilde{C_T} + w^*) \qquad (3.11) \\ \frac{\partial \pi_d^{s2}(q_s)}{\partial q_s^2} &= -\frac{1}{b} a^{1/b} z_0^{1/b} K_0[1 - F(z_0)] q_s^{-1/b-1} < 0 \qquad (3.12) \end{aligned}$$

Therefore $\pi_d^s(q_s)$ is concave in q_s, and the optimal order quantity $q_s^{**}(\alpha)$ that maximizes $\pi_d^s(q_s)$ is determined by setting equation 3.11 equal to zero where we get equation 3.9. After substituting the optimal order quantity $q_s^{**}(\alpha)$ into equation 3.8, we get equation 3.10. This completes the proof.

□

Until now, the optimal order quantity q_s^* can be summarized as follows:

$$q_s^* = \begin{cases} \bar{q}, & \text{if } q_s^{**}(\alpha) < \bar{q}; \\ q_s^{**}(\alpha), & \text{if } q_s^{**}(\alpha) > \bar{q}. \end{cases} \qquad (3.13)$$

3.1.3 Inducing Coordination

We have already figured it out that the optimal order quantity q_s^* will be equal to $q_1(\alpha)$, $\bar{q}$ or $q_s^{**}(\alpha)$. Now we take $q_s^* = q_1(\alpha)$ into consideration first. In this case, the optimal order quantity q_s^* is a function of the discount factor α, which here, we denotes as $q_s^*(\alpha)$. From equation 3.3, we get the optimal order quantity $q_s^*(\alpha)$ should satisfy

$$q_s^*(\alpha) = az_0[\frac{[1-F(z_0)]K_0}{w^* + C_T - 2\alpha q_s^*(\alpha)}]^b \tag{3.14}$$

by setting the first order condition equal to zero. Under the (TDI) scheme, the producer's objective function will be:

$$\pi_m^s = (w-c)q_s^*(\alpha) - \alpha q_s^*(\alpha)^2 \tag{3.15}$$

Without this scheme, the system turns to be a typical decentralized supply chain whose producer's profit is acquired from equation 2.12. Now we define a function $\Delta_m(\alpha)$, which stands for the producer's expected profit difference between the supply chain system without the scheme (equation 2.12) and with the scheme (equation 3.15).

$$\begin{aligned}\Delta_m(\alpha) &= (w^*-c)q_s^*(\alpha) - (w^*-c)az_0[\frac{[1-F(z_0)](b-1)K_0}{b(C_T+c)}]^b \\ &\quad -\alpha q_s^*(\alpha)^2\end{aligned} \tag{3.16}$$

When the whole supply chain gets coordinated, the optimal order quantity should be equal to the centralized supply chain order amount. By setting $q_s^*(\alpha) = q_c^*$, we can get the optimal α, denoted as α^*.

$$\alpha^* = \frac{c + C_T}{2q_c^*(b-1)} \tag{3.17}$$

Where q_c^* can be obtained from equation 2.14. So under the optimal discount α^*, the distributor's optimal order quantity will be equal to the centralized supply chain optimal order amount.

Lemma 3.4 *Under the optimal α^*, the producer acts better than the case without the TDI scheme.*

Proof α^* is the point that can make the optimal order quantity up to q_c^*. When we insert α^* into $\Delta_m(\alpha)$, we get:

$$\begin{aligned}
\Delta_m(\alpha^*) &= (w-c)q_s^*(\alpha^*) - (w-c)q_d^* - \alpha q_s^*(\alpha^*)^2 \\
&= (w-c)q_s^*(\alpha^*) - (w-c)az_0[\frac{[1-F(z_0)](b-1)K_0}{b(C_T+c)}]^b \\
&\quad -\alpha q_s^*(\alpha^*)^2 \\
&= \frac{c+C_T}{b-1}az_0[(1-F(z_0))K_0]^b(c+C_T)^{-b} \\
&\quad *[1-(\frac{b-1}{b})^b - \frac{1}{2}] \\
&= \frac{c+C_T}{b-1}az_0[\frac{(1-F(z_0))K_0}{c+C_T}]^b[\frac{1}{2}-(\frac{b-1}{b})^b]
\end{aligned}$$

We know that $\lim_{b\to\infty}(1+\frac{1}{-b})^{-b} = e$, then $\lim_{b\to\infty}(\frac{b-1}{b})^b = e^{-1}$. Meanwhile we find the first order condition of $(\frac{b-1}{b})^b$ is always larger than zero, so it is a monotonously increasing function. And when $b \to \infty$, $(\frac{b-1}{b})^b$ gets its maximum value which is $e^{-1} \approx 1/2.72$. Then $\Delta_m(\alpha^*)$ is always larger than zero. The producer acts better under the TDI scheme compared to the traditional decentralized supply chain system.

□

Lemma 3.5 *Under the optimal α^*, the distributor also acts better than the case without the TDI scheme.*

Proof Similar to lemma 3.4, we define $\Delta_d(\alpha)$ as the difference of the expected profit on the distributor side before and after using the TDI scheme.

$$\Delta_d(\alpha) = \frac{b}{b-1}a^{1/b}z_0^{1/b}K_0 q_s^{*1-1/b}[1-F(z_0)]-(C_T-\alpha q+w^*)q_s^*-\pi_d^* \tag{3.18}$$

After insert $q_s^*(\alpha) = q_c^*$, we get:

$$\Delta_d(\alpha) = \frac{q_c^*(c+C_T)}{b-1}[\frac{1}{2}-(\frac{b-1}{b})^{b-1}] \tag{3.19}$$

Following lemma 3.4, we can easily get $(\frac{b-1}{b})^{b-1} < \frac{1}{2}$, which makes $\Delta_d(\alpha) > 0$ for all $b > 1$. This illustrates that the TDI scheme makes the distributor earn more than the case without the scheme. The proof is completed.

□

Lemma 3.6 *Under the optimal α^* of the TDI scheme, the total supply chain profit will be equal to the centralized supply chain profit.*

Proof We define the total decentralized supply chain profit as the summation of the producer's and the distributor's profits. Under the transportation discount factor α^*, the total supply chain profit is described as follows.

$$\begin{aligned}
\pi_{total}^s &= \pi_m^s + \pi_d^s \\
&= (w^*-c)q_s^*(\alpha^*) + \frac{b}{b-1}a^{1/b}z_0^{1/b}q_s^*(\alpha^*)^{1-1/b}[1-F(z_0)]K_0 \\
&- q_s^*(\alpha^*)(C_T+w) \\
&= \frac{b}{b-1}a^{1/b}z_0^{1/b}q_c^{*1-1/b}[1-F(z_0)]K_0 - (c+C_T)q_c^* \\
&= q_c^*(c+C_T)\frac{1}{b-1} \\
&= \pi_c
\end{aligned}$$

Then the optimal α^* not only makes the optimal order quantity up to q_c^*, but also makes the total supply chain profit equal to the centralized system's profit.

□

For the case of the optimal order quantity equal to $\bar{q}$, we can simply let $\bar{q} = q_c^*$. And then, we can easily follow lemma 3.4, 3.5 and 3.6 to proof that, under the optimal discount factor α^*, the producer and distributor act better than before. Meanwhile, the total supply chain profit under the TDI scheme in this case is equal to the profit of the centralized system.

Through the previous two cases, we have already obtained that the supply chain can achieve coordination when $\alpha = \alpha^*$ and $\bar{q} = q_c^*$. The last case of the optimal order quantity is equal to $q_s^{**}(\alpha)$. By setting $q_s^{**}(\alpha) = q_c^*$, we get the optimal discount factor α^{**}.

$$\alpha^{**} = \frac{c + C_T}{\bar{q}(b-1)}$$

Lemma 3.7 *Under the optimal α^{**} of the TDI scheme, the producer has no motivation to offer the scheme to the distributor for the coordination given that $\bar{q} = q_c^*$.*

Proof Referring to equation 3.16, after we insert α^{**} and $\bar{q} = q_c^*$, we get:

$$\begin{aligned}
\Delta_m(\alpha) &= (w^* - c)q_s^{**}(\alpha) - (w^* - c)az_0[\frac{[1-F(z_0)](b-1)K_0}{b(C_T+c)}]^b \\
&\quad -\alpha q_s^{**}(\alpha)^2 \\
&= \frac{c+C_T}{b-q} az_0[1-F(z_0)]^b K_0^b (c+C_T)^{-b}[1-(\frac{b-1}{b})^b] \\
&\quad -\alpha q_s^{**}(\alpha)^2 \\
&= -(\frac{b-1}{b})^b < 0
\end{aligned}$$

The above result shows that the producer has to loss some potential profit in order to increase the distributor's optimal order quantity up to q_c^*. Then the producer has no motivation to do such coordination work. This completes the proof.

□

3.2 TDI Scheme II

Scheme II is based on the model discussed in chapter 2. We have known that the decentralized supply chain profit can not reach the centralized one. Scheme II is to provide two parts of compensation by the producer to the distributor:

1. A quantity loss and freshness level compensation contract, under which the producer shares portion of the quantity and freshness risk that the distributor has to face due to deterioration and decay of the product during the long distance transportation;

2. A compensation contract (similar to the traditional buy-back contract), under which the producer compensates the distributor certain amount for any unsold unit of the product.

Specifically, the products will arrive at the distributor side at time period t. The quantity and freshness level compensation contract suggests:

$$\beta(q,t) = \{w - c - \alpha_1[(az_0\theta(t))^{1/b}m(t)^{1-1/b}q^{-1/b} - c - C_T]\} \quad (3.20)$$

Where $\beta(q,t)$ is the quantity and freshness level compensation factor decided by the producer, and α_1 is a constant taking value in (0,1), which we will discuss later in this section. And this part of the compensation amount is as follows:

$$\beta(q,t) * q \quad (3.21)$$

Lemma 3.8 *The compensation factor $\beta(q,t)$ is an increasing function with respect to time period t and order quantity q.*

Proof The first order condition w.r.t q and t are as follows:

$$\frac{\partial\beta(q,t)}{\partial q} = \frac{1}{b}\alpha_1(az_0\theta(t))^{1/b}m(t)^{1-1/b}q^{-1/b-1} > 0 \quad (3.22)$$

$$\begin{aligned}\frac{\partial\beta(q,t)}{\partial t} &= -(az_0)^{1/b}q^{-1/b} \\ &*\{\frac{1}{b}\theta(t)^{1/b-1}m(t)^{1-1/b}\frac{d\theta(t)}{dt} \\ &+(1-1/b)\theta(t)^{1/b}m(t)^{-1/b}\frac{dm(t)}{dt}\} \quad (3.23)\end{aligned}$$

We have known that m(t) and $\theta(t)$ are both decreasing function w.r.t time period t. Then we get equation 3.23 is larger than zero, which indicates that $\beta(q,t)$ is an increasing function to t. And equation 3.22 shows the increasing property to q. This completes the proof.

□

Obviously, we have to ensure that the compensation amount from the producer to the distributor should be positive. Otherwise the compensation amount will be trivial. This is well demonstrated by the definition of the selling price based on the optimal stock factor. As known before,

$$p^* = [\frac{az_0\theta(t)}{q_d m(t)}]^{1/b}$$

then, the compensation amount can be rewritten as

$$\beta(q,t) = \{w - c - \alpha_1[(pm(t) - c - C_T]\}$$

When $m(t)$ is very small, the selling price will be extremely large to make it possible for the compensation to be positive. Meanwhile, for any unit of the product that the distributor has failed to sell in the market, the producer compensates the distributor an amount v as follows:

$$v = \alpha_1 p \tag{3.24}$$

Lemma 3.9 *Under the quantity and freshness level compensation and the compensation of equation 3.24, the distributor will order the quantity q_c^* for any $0 < \alpha_1 < 1$.*

Proof It is obvious that the optimal stock factor that maximizes $\pi_d(q,t)$ should be z_0. Then under the two parts of the TDI scheme II, the distributor's profit is as follows:

$$\begin{aligned}
\pi_d(q,t) &= pE_\epsilon[min(qm(t), D(p,t))] - wq - C_T q \\
&\quad +\beta(q,t)q + vE_\epsilon\{[qm(t) - D(p,t)]^+\} \\
&= pE_\epsilon[min(qm(t), D(p,t))] - wq - C_T q \\
&\quad +\{w - c - \alpha_1[(az_0\theta(t))^{1/b} m(t)^{1-1/b} q^{-1/b} - c - C_T]\}q \\
&\quad +vE_\epsilon\{[qm(t) - D(p,t)]^+\} \\
&= (1-\alpha_1)[pqm(t) - cq - C_T q] \\
&\quad -(1-\alpha_1)pE_\epsilon\{[qm(t) - D(p,t)]^+\} \\
&= (1-\alpha_1)\pi_c(q,t)
\end{aligned}$$

Thus, the optimal solution q_c^* that maximizes π_c optimizes π_d as well. That is, if the producer applies the two parts of the compensation scheme discussed above, the distributor will order up to q_c. Then the whole supply chain gets coordinated. This completes the proof.

□

3.3 TDI Scheme III

In scheme II, the whole supply chain gets coordinated through two different compensation contracts. Scheme III is to investigate whether we can achieve the coordination purpose through only one contract. The proposed new incentive scheme suggests that:

$$\begin{aligned}\Delta(q,t) &= w - c + \alpha_2\{c + C_T - (az\theta(t))^{1/b} q^{-1/b} m(t)^{1-1/b} \\ &\quad *[1 - F(z_0) + \int_0^{z_0} \frac{x}{z_0} f(x)dx]\}\end{aligned} \quad (3.25)$$

Where $\Delta(q,t)$ is the compensation amount offered by the producer for every unit of the product ordered by the distributor. α_2 is a constant taking value in (0,1). The TDI compensation only depends on the arrival time t and the order quantity q.

Lemma 3.10 *The compensation amount $\Delta(q,t)$ is an increasing function with respect to time period t and order quantity q.*

Proof The first order condition of $\Delta(q,t)$ w.r.t q and t are showed as follows:

$$\begin{aligned}\frac{\partial\Delta(q,t)}{\partial q} &= \frac{1}{k}\alpha_2(az_0\theta(t))^{1/b} q^{-1/b-1} m(t)^{1-1/b} \\ &\quad *[1 - F(z_0) + \int_0^{z_0} \frac{x}{z_0} f(x)dx] > 0\end{aligned} \quad (3.26)$$

$$\begin{aligned}\frac{\partial\Delta(q,t)}{\partial t} &= -\alpha_2(az_0)^{1/k} q^{-1/b}[1 - F(z_0) + \int_0^{z_0} \frac{x}{z_0} f(x)dx] \\ &\quad *\{\frac{1}{b}\theta(t)^{1/b-1}\frac{d\theta(t)}{dt} m(t)^{1-1/b} \\ &\quad +\theta(t)^{1/b}(1-\frac{1}{b})m(t)^{-1/b}\frac{dm(t)}{dt}\} > 0\end{aligned} \quad (3.27)$$

We have known that m(t) and $\theta(t)$ are both decreasing function w.r.t time period t. Then we can easily see that the above first order condition to t is larger than zero, which induces $\Delta(q,t)$ is increasing in t. Equation 3.26 also proves that $\Delta(q,t)$ is increasing in q. These complete the proof.

$\square$

Lemma 3.11 *Under the TDI compensation scheme described in equation 3.25, the distributor will order the quantity q_c^* for any $0 < \alpha_2 < 1$.*

Proof Under the TDI scheme III, the distributor's profit is as follows:

$$\begin{aligned}
\pi_d(q,t) &= pE_\epsilon[min(qm(t), D(p,t))] - wq - C_T q + \Delta(q,t)q \\
&= pE_\epsilon[min(qm(t), D(p,t))] - wq - C_T q \\
&\quad + \{w - c + \alpha_2\{c + C_T - (az\theta(t))^{1/b} q^{-1/b} m(t)^{1-1/b} * \\
&\quad [1 - F(z_0) + \int_0^{z_0} \frac{x}{z_0} f(x)dx]\}\}q \\
&= (1-\alpha_2)[pqm(t) - cq - C_T q] \\
&\quad -(az\theta(t))^{1/b} q^{-1/b} m(t)^{1-1/b} q\{[1 - \frac{\epsilon}{z_0}]^+ - \alpha_2 F(z_0) \\
&\quad + \alpha_2 \int_0^{z_0} \frac{x}{z_0} f(x)dx\} \\
&= (1-\alpha_2)[pqm(t) - cq - C_T q] \\
&\quad -(1-\alpha_2)pqm(t)E_\epsilon\{[1 - \frac{\epsilon}{z_0}]^+\} \\
&= (1-\alpha_2)\pi_c(q,t)
\end{aligned}$$

Thus, the optimal solution q_c^* that maximizes π_c optimizes π_d as well. That is, if the producer applies the compensation scheme discussed in equation 3.25, the distributor will order up to q_c. Then the whole supply chain gets coordinated. This completes the proof.

$\square$

Similar to TDI II, the compensation offered by the producer should be positive amount to the distributor. Based on the illustration of TDI II of the relationship between the optimal selling price and the marketable quantity factor $m(t)$, we can easily obtain the positive compensation amount.

□ **End of chapter.**

Chapter 4

Third-party Logistics

This chapter is to introduce a third-party logistics (3PL) provider to help the producer transport the products to the distributor side. The whole supply chain will include three parties. The optimal decisions faced by the 3PL provider, the producer and the distributor in the decentralized supply chain are studied respectively in this chapter. Based on the optimal decisions in the centralized supply chain as a benchmark, we propose a wholesale price policy between the producer and the distributor, and a transportation penalty factor offered by the 3PL provider to the distributor, to make the decentralized supply chain coordinated.

4.1 3PL Problem Formulation

We investigate the following problem. A producer transports by a third-party logistics provider a certain quantity of fresh product to a distant market. The product shipped has to undergo long distance transportation before it reaches the market. The transportation cost is paid by the distributor, which is compatible to the formulation discussed in chapter 2. The 3PL provider will set a guaranteed arrival time t_0 to the distributor. If the product can not reach the distributor side within t_0, the 3PL provider will pay certain amount of the penalty. The penalty is related to the transportation cost per unit which takes the following form:

$$C_T - \alpha_3(t - t_0)^+ \tag{4.1}$$

Where α_3 is the penalty factor describing the amount the 3PL provider has to pay when the transportation time delays. For the producer and distributor, all the parameters are consistent with chapter 2. For our convenience, table 4.1 lists all the notation in this chapter.

Without loss of any generality, we assume the salvage value of any product left unsold is zero since it is highly perishable. Also we do not consider any shortage cost. All the information is assumed to be common knowledge and all the decision makers are risk-neutral with the objective to maximize their expected profit. To proceed, we summarize the business sequence in detail. (1)The producer will offer a wholesale price to the distributor and 3PL provider; (2)Based on the wholesale price, the 3PL provider will propose a guaranteed arrival time; (3)The distributor determines the purchasing quantity and selling price after he obtains the wholesale price and the guaranteed arrival time.

The optimization problems of the distributor, 3PL provider, and the producer are listed as follows:

- The distributor's decision includes the selling price and the purchasing quantity. After obtaining the guaranteed arrival time t_0 from the 3PL provider and the wholesale price from the producer, the distributor's purpose is to maximize its own expected profit $\pi_d(p, q_d|t_0, w)$:

$$\begin{aligned} \pi_d(p, q_d|t_0, w) &= pE_{t,\epsilon}\{min[q_d m(t), D(p,t)]\} - wq_d \\ &\quad - E_t\{C_T - \alpha_3(t - t_0)^+\}q_d \end{aligned} \tag{4.2}$$

Table 4.1: List of notation.

Symbol	*Description*
τ	Fresh duration of the product;
t	The transportation time distributed over [j,k], follow uniform distribution with PDF g(.) and CDF G(.);
$\theta(t)$	Freshness index of the product, with transportation time t;
m(t)	Marketable quantity of the product, with transportation time t;
b	The price elasticity of the market demand;
ϵ	Random fluctuations of the demand, with PDF f(.) and CDF F(.);
p	The selling price set by the distributor, *a decision variable*;
D(p,t)	The market demand when selling price is p and freshness level is $\theta(t)$;
c	Unit manufacturing cost of the producer;
c_1	Unit transportation cost of the 3PL provider;
α_3	The penalty factor offered by the 3PL provider;
C_T	Unit fixed transportation cost of the 3PL provider;
q_d	The shipping quantity of the distributor, *a decision variable*;
t_0	The guaranteed arrival time set by the 3PL provider, *a decision variable*;
w	The wholesale price offered by the producer, *a decision variable*;
$\pi_{3PL}(.)$	The expected profit function of the 3PL provider;
$\pi_m(.)$	The expected profit function of the producer;
$\pi_d(.)$	The expected profit function of the distributor;
q_c	The shipping quantity in the centralized supply chain;
p_c	The selling price in the centralized supply chain;
π_c	The expected entire chain profit in the centralized supply chain.

where q_d is the purchasing quantity, which is influenced by the wholesale price w and the guaranteed arrival time t_0.

- Considering the 3PL provider who needs to determine the guaranteed arrival time t_0, its goal is to maximize the expected profit $\pi_{3PL}(t_0, q_d(w, t_0))$:

$$\pi_{3PL}(t_0, q_d(w, t_0)) = E_t\{C_T - \alpha_3(t - t_0)^+ - c_1\} * q_d(w, t_0) \tag{4.3}$$

  Where $q_d(w, t_0)$ is the shipping quantity of the 3PL provider, which is influenced by the wholesale price and the guaranteed arrival time t_0.

- The producer needs to determine the wholesale price with the objective to maximize its expected profit $\pi_m(w)$:

$$\pi_m(w, q_d(w)) = (w - c)q_d(w) \tag{4.4}$$

  Where $q_d(w)$ is the order quantity of the distributor when the 3PL provider has set the guaranteed arrival time t_0.

4.2 Optimal Decisions in the Decentralized Supply Chain

In this section, we will characterize the optimal decisions of the 3PL provider, the producer and the distributor in the decentralized supply chain in which they seek to maximize their respective profits. We will do so by using a backwards approach. First, we will derive the joint decisions of the distributor, given the transportation time, the arbitrary wholesale price and the guaranteed arrival time. Second, we analyze the 3PL provider's optimal guaranteed arrival time, given any wholesale price and the transportation time. Finally, based on the optimal decisions of the distributor and the 3PL provider, we derive the optimal wholesale price.

4.2.1 Optimal Decisions of the Distributor

The distributor faces a joint purchasing quantity and pricing decision problem, given that the transportation time is t and the wholesale price w. From chapter 2, we have already obtained the stock factor z where

$$z = \frac{qm(t)}{ap^{-b}\theta(t)} \tag{4.5}$$

Then the problem of optimizing (q_d, p) is converted into that of optimizing (q_d, z). By inserting equation 4.5 into 4.2, the distributor's objective function can be rewritten as:

$$\begin{aligned}\pi_d(p, q_d|t_0) &= (\frac{az\theta(t)}{qm(t)})^{1/b} E_{t,\epsilon}\{min[q_d m(t), D(p,t)]\} \\ &\quad -wq_d - E_t\{C_T - \alpha_3(t-t_0)^+\}q_d\end{aligned}$$

The optimal stock factor z_0 obtained in lemma 2.1 can also be applied to this model since the change of the transportation cost does not distort z. While through z_0, we get $p^* = (\frac{az\theta(t)}{qm(t)})^{1/b}$. After substituting p^* and z_0 into 4.2, we get:

$$\begin{aligned}\pi_d(q_d) &= \frac{b}{b-1}(az_0)^{1/b} K_0 q_d^{1-1/b}[1-F(z_0)] \\ &\quad -wq_d - E_t\{C_T - \alpha_3(t-t_0)^+\}q_d \qquad (4.6)\end{aligned}$$

Where $K_0 = E_t\{\theta(t)^{1/b} m(t)^{1-1/b}\}$.

Lemma 4.1 *For any given wholesale price w of the producer, the distributor's optimal order quantity should be:*

$$q_d^* = az_0\{\frac{1-F(z_0)}{w + C_T - \alpha_3 E_t\{[t-t_0]^+\}} K_0\}^b \tag{4.7}$$

Proof The first and second order condition are as follows

$$\begin{aligned}\frac{\partial \pi_d(q_d)}{\partial q_d} &= (az_0)^{1/b} K_0 q_d^{-1/b}[1 - F(z_0)] - w \\ &\quad - E_t\{C_T - \alpha_3[t - t_0]^+\} \\ \frac{\partial^2 \pi_d(q_d)}{\partial {q_d}^2} &= -\frac{1}{b}(az_0)^{1/b} K_0 q_d^{-1/b-1}[1 - F(z_0)] < 0\end{aligned}$$

Therefore $\pi_d(q_d)$ is concave in q_d, and the optimal order quantity q_d^* that maximizes $\pi_d(q_d)$ is determined by the first order condition from which 4.7 is obtained.

□

Remark 1

1. q_d^* decreases in the producer's wholesale price w.
2. Since t follows uniform distribution in [j, k], we can rewrite equation 4.7 as:

$$q_d^* = az_0\{\frac{1 - F(z_0)}{w + C_T - \frac{\alpha_3(t_0-k)^2}{2(k-j)}} K_0\}^b \tag{4.8}$$

   Through the first order condition of 4.8, we get

$$\begin{aligned}\frac{\partial q_d^*(t_0, w)}{\partial t_0} &= az_0 b[1 - F(z_0)]^b K_0^b (w + C_T - \frac{\alpha_3(t_0 - k)^2}{2(k - j)})^{-b-1} \\ &\quad * \frac{\alpha_3}{k - j}(t_0 - k)\end{aligned}$$

   Then we can easily see that the optimal order quantity q_d^* decreases in t_0 within [j, k].

4.2.2 Optimal Decisions of the 3PL Provider

We have already got the optimal order quantity q_d^* in equation 4.8, after insert the q_d^* into equation 4.3, we get:

$$\pi_{3PL}(t_0) = [C_T - c_1 - \frac{\alpha_3(t_0 - k)^2}{2(k - j)}]az_0\{\frac{1 - F(z_0)}{w + C_T - \frac{\alpha_3(t_0-k)^2}{2(k-j)}} K_0\}^b \tag{4.9}$$

Lemma 4.2 *Based on the 3PL provider's objective function 4.9, the optimal guaranteed arrival time t_0^* that maximizes the its own expected profit is:*

$$t_0^* = k - \sqrt{\frac{2(k-j)[(b-1)C_T - bc_1 - w]}{\alpha_3(b-1)}} \tag{4.10}$$

Proof Taking the first derivative with respect to t_0, we have

$$\begin{aligned}
\frac{\partial \pi_{3PL}(t_0)}{\partial t_0} &= az_0 \frac{\alpha_3(t_0-k)}{k-j}\{[1-F(z_0)]K_0\}^b \\
&\quad *[w + C_T - \frac{\alpha_3(t_0-k)^2}{2(k-j)}]^{-b-1} \\
&\quad \{(1-b)\frac{\alpha_3(t_0-k)^2}{2(k-j)} - w - C_T + b(C_T - c_1)\} \\
&= az_0 \frac{\alpha_3(t_0-k)}{k-j}\{[1-F(z_0)]K_0\}^b \\
&\quad *[w + C_T - \frac{\alpha_3(t_0-k)^2}{2(k-j)}]^{-b-1} \\
&\quad \frac{(1-b)\alpha_3}{2(k-j)}[t_0 - k - \sqrt{\Omega(w)}][t_0 - k + \sqrt{\Omega(w)}]
\end{aligned}$$

Where $\Omega(w) = \frac{2[(b-1)C_T - w - bc_1](k-j)}{(b-1)\alpha_3}$.

Apparently, $\pi_{3PL}(t_0)$ increases before t_0 reaches

$$k - \sqrt{\frac{2(k-j)[(b-1)C_T - bc_1 - w]}{\alpha_3(b-1)}}$$

and starts to decrease after that point. Then the optimal guaranteed arrival time t_0^* is obtained. This completes the proof.

□

Remark 2

1. To ensure that the optimal t_0^* can be obtained, one important assumption should be notified.

$$w < (b-1)C_T - bc_1 \tag{4.11}$$

   This makes t_0^* is valid.

2. t_0^* increases in the wholesale price w. This illustrates that if the product is expensive, the 3PL provider will set the guaranteed arrival time higher to decrease his own risk for the delay of the transportation.

3. Inserting the optimal t_0^* into the optimal order quantity 4.8, we get

$$q_d^* = az_0\{\frac{1-F(z_0)}{w+c_1}K_0\}^b[\frac{b-1}{b}]^b \tag{4.12}$$

4.2.3 Optimal Decisions of the Producer

Referring to the objective function of the producer introduced in equation 4.4, after inserting the optimal order quantity q_d^* of equation 4.12, we get

$$\pi_m(w) = (w-c)az_0\{\frac{1-F(z_0)}{w+c_1}K_0\}^b[\frac{b-1}{b}]^b \tag{4.13}$$

Lemma 4.3 *The producer's optimal wholesale price w^* is*

$$w^* = \frac{bc+c_1}{b-1} \tag{4.14}$$

Proof Taking the first derivative with respect to w, we have

$$\begin{aligned}\frac{\partial \pi_m(w)}{\partial w} &= az_0\{[1-F(z_0)]K_0\}^b(\frac{b-1}{b})^b(w+c_1)^{-b-1} \\ &\quad *[(1-b)w+c_1+bc] \end{aligned} \tag{4.15}$$

Obviously, we can see that $\pi_m(w)$ increases before w reaches $\frac{bc+c_1}{b-1}$ and starts to decrease after this point. Therefore, the optimal wholesale price that maximizes $\pi_m(w)$ is $\frac{bc+c_1}{b-1}$. This completes the proof.

□

After obtaining the optimal wholesale price w^*, we insert it into the optimal order quantity 4.12 and guaranteed arrival time 4.10. Then we get:

$$q_d^* = az_0\{\frac{1-F(z_0)}{c+c_1}K_0\}^b[\frac{b-1}{b}]^{2b} \tag{4.16}$$

$$t_0^* = k - \sqrt{\frac{2(k-j)[(b-1)C_T - bc_1 - \frac{bc+c_1}{b-1}]}{\alpha_3(b-1)}} \tag{4.17}$$

Remark 3

1. w^* is greater than c because $b > 1$, which guarantees that the producer always earns a positive profit.

2. w^* is decreasing in b, which implies that the producer should decrease his wholesale price if the market demand is more price-sensitive.

3. Referring to the assumption illustrated in equation 4.11, after inserting the optimal wholesale price w^*, we get:

$$bc + c_1(b^2 - b + 1) < (b-1)^2 C_T \tag{4.18}$$

Only when equation 4.18 satisfies does the optimal guaranteed arrival time t_0^* exist.

4. We can derive the optimal expected profit for the producer, distributor and the 3PL provider based on the optimal decisions obtained in section 4.2.

   - The producer's optimal expected profit is:

$$\pi_m^* = \frac{c+c_1}{b-1} * az_0\{\frac{1-F(z_0)}{c+c_1}K_0\}^b[\frac{b-1}{b}]^{2b} \tag{4.19}$$

   - The distributor's optimal expected profit is:

$$\pi_d^* = (\frac{b}{b-1})^2\frac{c+c_1}{b-1} * az_0\{\frac{1-F(z_0)}{c+c_1}K_0\}^b[\frac{b-1}{b}]^{2b} \tag{4.20}$$

   - The 3PL provider's optimal expected profit is:

$$\pi_{3PL}^* = \frac{b(c+c_1)}{(b-1)^2} * az_0\{\frac{1-F(z_0)}{c+c_1}K_0\}^b[\frac{b-1}{b}]^{2b} \tag{4.21}$$

   Therefore, the relative profits of the three parties in the supply chain are as follows:

$$\pi_m : \pi_d : \pi_{3PL} = 1 : (\frac{b}{b-1})^2 : \frac{b}{b-1} \tag{4.22}$$

   Since $b > 1$, the ratio of the relative profits only depends on the price elasticity b. And also from the ratio, we know that $\pi_m < \pi_{3PL} < \pi_d$. That is, without coordination, the distributor achieves the largest portion of the total chain welfare, whereas the producer achieves the smallest portion, especially when the price elasticity is small.

4.3 Optimal Decisions in the Centralized Supply Chain

The centralized supply chain, we mean the three parties of the decentralized supply chain are owned by one company, and there exists a central decision maker who seeks to achieve the maximal expected profit of the entire supply chain. In that case, only two decisions need to be made. In the first stage, the order quantity to the distant market should be determined; and in the second stage, when the product arrives at the destination, the selling price which depends on the freshness level and quantity loss should be determined. To be different from the decentralized supply chain, we denote the order quantity and the selling price in the centralized system as q_c and p_c. The profit function, denoted as π_c becomes

$$\pi_c(q_c) = p_c E_{\epsilon,t}\{min[qm(t), D(p_c, t)]\} - cq_c - c_1 q \tag{4.23}$$

Lemma 4.4 *In the centralized supply chain, the optimal order quantity q_c^* is given by*

$$q_c^* = az_0[\frac{1-F(z_0)}{c+c_1}K_0]^b \tag{4.24}$$

And the corresponding optimal expected profit π_c^* is

$$\pi_c^* = \frac{c+c_1}{b-1} * az_0[\frac{1-F(z_0)}{c+c_1}K_0]^b \tag{4.25}$$

Proof From equation 2.1, we can easily get

$$p_c^*(q_c, t) = (\frac{az_0\theta(t)}{q_c m(t)})^{1/b} \tag{4.26}$$

Inserting 4.26 into 4.23, we transform the profit as the function of the order quantity q_c.

$$\pi_c(q_c) = \frac{b}{b-1}(az_0)^{1/b} K_0 q_c^{1-1/b}[1-F(z_0)] - cq_c - c_1 q_c \tag{4.27}$$

Apparently, π_c is concave in q_c since $b > 1$. Therefore, the optimal order quantity is uniquely determined by the first order condition. Let

$$(az_0)^{1/b} K_0 q_c^{-1/b}[1-F(z_0)] - c - c_1 = 0 \tag{4.28}$$

we get the optimal order quantity q_c^* given by equation 4.24. After substituting equation 4.24 into 4.27, we have equation 4.25. This completes the proof.

□

Remark 4

1. The optimal order quantity in the centralized system is larger than that in the decentralized system:

$$q_c^* : q_d^* = (\frac{b}{b-1})^{2b} \tag{4.29}$$

This shows that the ration only depends on the price elasticity of the market demand. Also $(\frac{b}{b-1})^{2b}$ is decreasing in $b \in (1, +\infty)$. Therefore, the more sensitive the market demand to the selling price, the closer the optimal order quantity in the centralized supply chain to the decentralized one.

2. We define the total expected profit of the decentralized supply chain is $\pi_m^* + \pi_d^* + \pi_{3PL}^*$. Denote $\varsigma = 1 - (\pi_m^* + \pi_d^* + \pi_{3PL}^*)/\pi_c^*$ as the expected profit loss due to the lack of coordination. Hence we have

$$\begin{aligned} \varsigma &= 1 - \frac{\pi_m^* + \pi_d^* + \pi_{3PL}^*}{\pi_c^*} \\ &= 1 - (\frac{b-1}{b})^{2b} - (\frac{b-1}{b})^{2b-2} - (\frac{b-1}{b})^{2b-1} \end{aligned} \tag{4.30}$$

Through figure 4.1, we know that the profit loss, ς, is increasing in k. That is, the more sensitive the market demand is to a change in price, the more profit loss will be incurred due to the lack of coordination.

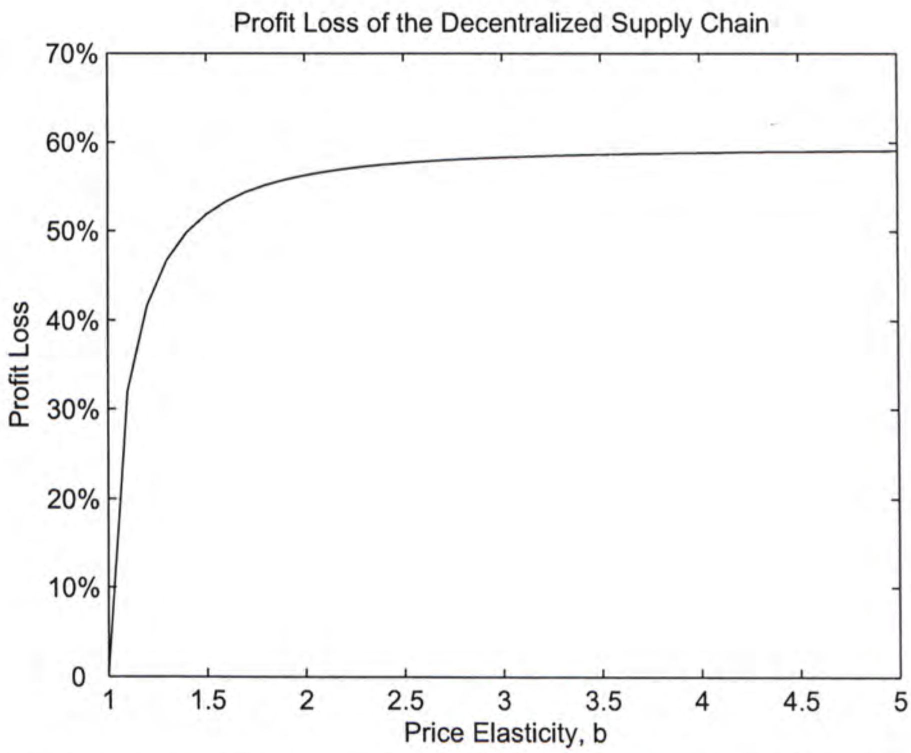

Figure 4.1: Relative Profit Loss between the Centralized and the Decentralized System

4.4 Designing a Coordination Mechanism

To make the decentralized fresh product supply chain coordinated, two objectives should be reached: (1) the total profit of the 3PL provider, the producer and the distributor under the optimal decisions should be the same as that of the centralized system. (2) each party should be better off so that they are willing to participate in the coordination. In our model, we notice that in the decentralized supply chain, the distributor gets the largest amount of the profit. While the producer get the smallest part. Therefore, to motivate the distributor order more products, the producer should offer incentive since the producer's expected profit is determined from the estimation of the order quantity of the distributor. Meanwhile, the 3PL provider needs to give the distributor some allowance since the transportation cost is paid by the distributor. And the allowance will encourage the distributor to order more so as to increase the 3PL provider's expected profit. Our proposed contracts will run in a sequential order: first the producer negotiates with the distributor and a wholesale pricing policy is determined; then the distributor negotiates with the 3PL provider so as to determine the penalty factor policy. Specifically:

(1) Our wholesale pricing policy suggests that the distributor agrees to purchase all the producer's marketable quantity at $w(q,t)$, which takes the following form:

$$\begin{aligned} w(q,t) &= c+\beta\{(\frac{az_0\theta(t)}{q})^{1/b}m(t)^{1-1/b}min[1,\frac{\epsilon}{z_0}] \\ &\quad -c-c_1\} \end{aligned} \tag{4.31}$$

Where q is the order quantity of the distributor, t is the realized transportation time, and β is a constant within (0,1) which is determined in the negotiation process between the producer and the distributor.

(2) Our penalty factor policy suggests that the 3PL provider will offer some transportation allowance depending on the order quantity and the transportation time period. The penalty factor will take the following form:

$$\begin{aligned}\alpha_3(q,t) &= \frac{C_T - c_1}{(t-t_0)^+} \\ &-\frac{(1-\beta)\gamma}{(t-t_0)^+}\{(\frac{az_0\theta(t)}{q})^{1/b}m(t)^{1-1/b}min[1,\frac{\epsilon}{z_0}] \\ &-(c+c_1)\} \qquad (4.32)\end{aligned}$$

Where γ is a constant taking value in (0, 1), which is determined by the negotiation process between the distributor and the 3PL provider. We will do further analysis later in this section about the constant β and γ.

Remark 5

1. we can easily see that w(q,t) is strictly decreasing in the order quantity q for any t. This is similar to the traditional Quantity Discount where the wholesale price is decreasing in the distributor's purchasing quantity. Meanwhile, for any fixed q, w(q,t) is strictly decreasing in t. Therefore, if the transportation time becomes longer, the producer has to reduce his wholesale price to encourage the distributor to order more.

2. The penalty factor policy $\alpha_3(q,t)$ takes the following form when t follows the uniform distribution within [j, k].

$$\begin{aligned}\alpha_3(q,t) &= \{\frac{C_T - c_1}{(k-t_0)^2} \\ &-\frac{(1-\beta)\gamma}{(k-t_0)^2}\{(\frac{az_0\theta(t)}{q})^{1/b}m(t)^{1-1/b}min[1,\frac{\epsilon}{z_0}] \\ &-(c+c_1)\}\} * 2(k-j)\end{aligned}$$

Obviously, we can see that the penalty factor α_3 is increasing in the transportation time t for any order quantity. So the longer the shipping time, the more penalty the 3PL has to pay if the product does not arrive at the distributor side on time. Meanwhile, α_3 is strictly increasing in q for any given time period t. Therefore, the 3PL provider is willing to take more penalty risk for the delay of the product so that the distributor can order more.

Theorem 4.1 *The wholesale pricing policy 4.31 together with the penalty factor policy 4.32 will induce the decentralized supply chain achieve the same performance as that of the centralized supply chain for any β, $\gamma \in (0,1)$. Meanwhile, for the decentralized supply chain under 4.31 and 4.32, the expected profit shares of the 3PL provider, the producer and the distributor are $(1-\beta)\gamma\pi_c^*$, $\beta\pi_c^*$ and $(1-\beta)(1-\gamma)\pi_c^*$.*

Proof It is easily to see that the optimal stock factor that maximizes the distributor's expected profit should be z_0 (refer to lemma 2.1). Therefore, after inserting equation 4.31 to the distributor's optimal profit function $\pi_d^{'}$, we have

$$\begin{aligned}
\pi_d^{'}(q,t) &= pE_\epsilon\{min[qm(t), D(p,t)] - w(q,t)q \\
&\quad -E_t\{C_T - \alpha_3(q,t)(t-t_0)^+\}q \\
&= (1-\gamma)(1-\beta)[(\frac{az_0\theta(t)}{q})^{1/b}m(t)^{1-1/b}min[1,\frac{\epsilon}{z_0}] - c - c_1]q \\
&= (1-\beta)(1-\gamma)\pi_c^*
\end{aligned}$$

Next we investigate the producer's optimal profit $\pi_m^{'}$. Based on the wholesale pricing policy from equation 4.31 and the penalty factor of equation 4.32, we can easily get the following:

$$\begin{aligned}
\pi_m^{'}(q,t) &= (w(q,t)-c)q \\
&= \beta[(\frac{az_0\theta(t)}{q})^{1/b}m(t)^{1-1/b}min[1,\frac{\epsilon}{z_0}] - c - c_1]q \\
&= \beta\pi_c^*
\end{aligned}$$

Finally, the 3PL provider's optimal profit π'_{3PL} will be obtained through the penalty factor policy from equation 4.32.

$$\begin{aligned}\pi'_{3PL} &= \{C_T - \alpha_3(q,t)(t-t_0)^+ - c_1\}q \\ &= (1-\beta)\gamma\pi_c^*\end{aligned}$$

Therefore the order quantity that maximizes π'_d, π'_m and π'_{3PL} equals to the optimal shipping quantity of the centralized supply chain q_c^*. This completes the proof.

□

Until now, we know that the profit shares among the 3PL provider, the producer and the distributor are determined by the values of β and γ. Through negotiation, the producer's profit share (β) is first determined, and the rest share (1-β) goes to the distributor and the 3PL provider. After that, the distributor's and the 3PL provider's profit share are determined. Generally speaking, the values of β and γ are determined by the relative bargaining powers of the three parties involved in the supply chain. However, to ensure that each party is willing to participate in the coordination, a fundamental requirement is that each party will be better off. This results in a upper and a lower bound on β and γ, as follows:

$$\begin{aligned}(\frac{b-1}{b})^{2b} < \ &\beta < 1 - (\frac{b-1}{b})^{2b-1} - (\frac{b-1}{b})^{2b-2} \\ \frac{1}{1-\beta}(\frac{b-1}{b})^{2b-1} < \ &\gamma < 1 - \frac{1}{1-\beta}(\frac{b-1}{b})^{2b-2}\end{aligned}$$

As can be seen from figure 4.2, the value of γ that is acceptable to both the 3PL provider and the producer depends on the value of β. When β grows, the acceptable range of γ becomes narrower. This is due to the fact that the sharable profit $(1-\beta)\pi_c^*$ shrinks.

□ **End of chapter.**

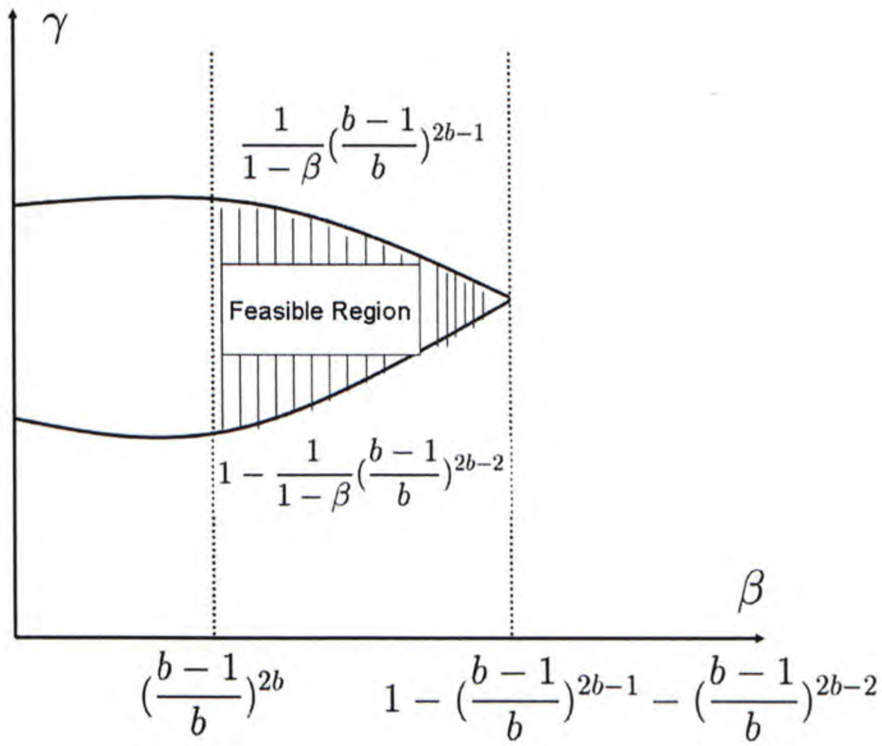

Figure 4.2: The Lower and Upper Bounds of β and γ

Chapter 5

Computational Studies

5.1 Numerical Study of the 3PL Model

We have conducted a series of computational study to evaluate the effect of different parameters to the optimal decisions. The numerical study can uncover the managerial insights that are not clear in the theoretical results. The experiment will focus on the effects of the fresh duration and the price elasticity b. Meanwhile, we also compare the two coordination mechanisms of TDI III and 3PL one which includes a wholesale pricing policy and the penalty factor policy. The parameters are defined as follows: the unit production cost $c = 2$ and the unit shipping cost $c_1 = 1$. And the unit transportation cost paid by the distributor $C_T = 8$ and the penalty factor $\alpha_3 = 2$. The transportation time t is assumed to follow uniform distribution in [1,5]. The product decays through a constant exponential function $m(t) = e^{-0.1(t-\tau)}$ and the value drop function $\theta(t) = e^{-0.2(t-\tau)}$. In the demand function, $a = 100$ and the price elasticity $b = 2$. The random factor ϵ follows a normal distribution with mean 1 and variance 0.2.

In the first group of the experiment, we change the fresh duration from 2 to 4. For each τ, we list the optimal decisions for both the decentralized and centralized supply chain. At the same time, we evaluate the profit loss and profit margin through different fresh durations. The results are showed in table 5.1.

Table 5.1: Optimal Decisions under Different Fresh Durations

τ	w^*	q_d^*	t_0^*	π_m^*	π_d^*	π_{3PL}^*	q_c^*	π_c^*	ς
2.00	5.00	0.71	3.00	2.13	8.52	4.26	11.36	34.07	56.25%
2.25	5.00	0.77	3.00	2.30	9.18	4.59	12.24	36.73	56.25%
2.50	5.00	0.82	3.00	2.47	9.90	4.95	13.20	39.59	56.25%
2.75	5.00	0.89	3.00	2.67	10.67	5.33	14.22	42.67	56.25%
3.00	5.00	0.96	3.00	2.87	11.50	5.75	15.33	46.00	56.25%
3.25	5.00	1.03	3.00	3.10	12.39	6.20	16.53	49.58	56.25%
3.50	5.00	1.11	3.00	3.34	13.36	6.68	17.81	53.44	56.25%
3.75	5.00	1.20	3.00	3.60	14.40	7.20	19.20	57.60	56.25%
4.00	5.00	1.29	3.00	3.88	15.52	7.76	20.70	62.09	56.25%

Observation I

- Through table 5.1, we see that the order quantities, the expected profit for both the decentralized supply chain and centralized supply chain change in the same direction as the fresh duration. This is because the longer the fresh duration τ is, the larger the remaining profit of the fresh product is after the long distance transportation.

- From Table 5.1, we obtain that the profit loss will remain the same no matter how long the fresh duration τ is. This is due to our risk neutral assumption on the involved parties. We conjecture that if considering the risk averse setting, the profit loss should be decreasing in the fresh duration.

- According to table 5.1, the wholesale price, the guaranteed arrival time and the profit loss are unchanged with regard to the fresh duration since the price elasticity b is fixed. For the guaranteed arrival time, when the fresh duration is short, the guaranteed arrival time may be larger than the fresh duration. This states that the 3PL provider can only guarantee the product's arrival with certain level of profit loss and value drop.

The second group of experiment focus on the price elasticity. We change b from 2 to 3.4, and the optimal decisions for different parties in both decentralized and centralized system are summarized in table 5.2.

Table 5.2: Optimal Decisions under Different Price Elasticities

b	w^*	q_d^*	t_0^*	π_m^*	π_d^*	π_{3PL}^*	q_c^*	π_c^*	q_c^*/q_d^*	ς
2.00	5.00	0.96	3.00	2.87	11.50	5.75	15.33	46.00	16.00	56.25%
2.20	4.50	1.07	1.89	2.68	8.99	4.91	15.41	38.53	14.40	56.97%
2.40	4.14	1.15	1.35	2.47	7.25	4.23	15.31	32.80	13.29	57.47%
2.60	3.88	1.20	1.02	2.24	5.92	3.64	14.93	27.99	12.49	57.83%
2.80	3.67	1.25	0.80	2.08	5.03	3.24	14.82	24.70	11.87	58.10%
3.00	3.50	1.26	0.64	1.89	4.25	2.83	14.34	21.51	11.39	58.30%
3.20	3.36	1.28	0.52	1.74	3.68	2.53	14.05	19.16	11.00	58.46%
3.40	3.25	1.27	0.43	1.59	3.20	2.26	13.62	17.02	10.68	58.59%

Observation II

- As the market demand becomes more sensitive to the selling price, the wholesale price and the optimal order quantity will decrease significantly because of the lack of coordination.

- The guaranteed arrival time is decreasing with respect to the b, this implies that the 3PL provider is eager to propose the shipping contract to the distributor with a shorter guaranteed arrival time when the market demand becomes more price sensitive. This can be seen as a kind of motivation to encourage the distributor using the 3PL provider to ship the product.

- The expected profits of all the parties for both the decentralized and centralized supply chain do strictly decrease in b. Hence, all of them should prefer a less price-sensitive market demand.
- The profit loss is increasing in b because of the market competition among the three parties. Therefore, the more sensitive the demand is to the selling price, the more beneficial the coordination of the three parties will be.

5.2 Comparison between the Supply Chain with and without the 3PL provider

In this section, we try to compare whether the distributor should select the 3PL provider to help him transport the product or just deliver the product by himself. Through the economy of scale, the 3PL provider normally will provide the transportation service better than the distributor himself. The 3PL provider's advantage mainly shows in two aspects. One is that the cost of transportation through the 3PL provider is lower than that of the distributor himself. The other is that the delivery time is shorter by the 3PL provider. Since that the 3PL provider needs to earn a portion of profit through the transportation service, his shipping price charged to the distributor should be higher than the shipping cost of the distributor himself. Through our computational study, we assume that the mean of the delivery time by the 3PL provider is 20% shorter than the distributor's transportation. Also, the cost of 3PL provider is 20% less than the distributor himself. Equivalently, we set t as the delivery time for the 3PL provider. So the time for shipment by the distributor himself would be 0.8t. Also we set $C_T = 4$ as the transportation cost by the distributor himself. Then the cost of 3PL provider c_1 would be 3.2. The price of the transportation service by the 3PL provider $\widehat{C_T}$ should satisfy $\widehat{C_T} > C_T$. The computational study will be investigated through two aspects: the freshness level, quantity loss and the price elasticity index. The factor that we evaluate is the increase or decrease ratio with respect to the distributor's expected profit. For example, under the TDI scheme, the distributor will get x amount of the profit through delivering the product by himself. And in the 3PL model, the distributor will get y amount of the profit. We define ϖ as the increase or decrease ratio of the distributor's expected profit between the TDI scheme and the 3PL model.

$$\varpi = \frac{x - y}{x}$$

Figure 5.3 shows the effect of ϖ when the freshness level and the quantity loss level change. Recall that we have assumed that the freshness level and quantity loss are followed by the exponential distribution. To facilitate our study, we now set the freshness level and quantity loss function as follows:

$$\theta(t) = e^{-o_1(t-\tau)}$$

$$m(t) = e^{-o_2(t-\tau)}$$

Observation III

- From table 5.3, we can see that as the factor o_1 and o_2 become larger, which means the product has a significant rate of the freshness level decay or quantity loss, the distributor's decreasing ratio starts to decrease. This is because that when the product gets decay or deterioration seriously, the 3PL's shipping advantages (e.g. less shipping time and lower shipping cost) become more efficient. Therefore, the

Table 5.3: The comparison with respect to the freshness level and quantity loss

		TDI Scheme			3PL Model				
o_1	o_2	π_c^*	π_d^*	π_m^*	π_c^*	π_d^*	π_m^*	π_{3PL}^*	ϖ
0.20	0.10	2.17	1.09	0.54	2.97	0.74	0.19	0.37	31.67%
0.24	0.10	2.10	1.05	0.53	2.94	0.73	0.18	0.37	30.22%
0.28	0.10	2.04	1.02	0.51	2.91	0.73	0.18	0.36	28.77%
0.32	0.10	1.98	0.99	0.50	2.88	0.72	0.18	0.36	27.31%
0.36	0.10	1.93	0.96	0.48	2.86	0.71	0.18	0.36	25.86%
0.40	0.10	1.87	0.94	0.47	2.83	0.71	0.18	0.35	24.39%
0.44	0.10	1.83	0.91	0.46	2.81	0.70	0.18	0.35	22.93%
0.48	0.10	1.78	0.89	0.44	2.79	0.70	0.17	0.35	21.47%
0.52	0.10	1.74	0.87	0.43	2.78	0.69	0.17	0.35	20.01%
0.20	0.15	2.09	1.04	0.52	2.93	0.73	0.18	0.37	29.86%
0.20	0.25	1.94	0.97	0.48	2.86	0.72	0.18	0.36	26.22%
0.20	0.35	1.81	0.91	0.45	2.81	0.70	0.18	0.35	22.57%
0.20	0.45	1.71	0.85	0.43	2.77	0.69	0.17	0.35	18.91%
0.20	0.55	1.62	0.81	0.40	2.74	0.68	0.17	0.34	15.26%
0.20	0.65	1.54	0.77	0.38	2.72	0.68	0.17	0.34	11.62%

3PL provider should be a better choice to the distributor if the product has serious value drop as time goes.

- Another important insight through table 5.3 is that the distributor's expected profit will drop after using 3PL provider to ship the product. This relates to the original set up cost which we did not consider when doing the modeling in chapter 3. Since if the distributor does the shipment by himself, the set up cost actually will be a large cost which affect the expected profit. Then, whether the distributor's expected profit decreases or increases, the set up cost should be included. If the result of the distributor's expected profit deducting the set up cost is less than the expected profit obtained in the 3PL model, the 3PL provider should be the better choice to the distributor for the transportation services. Otherwise, the distributor will prefer to do the shipment by himself.

- Some extreme cases are also worthy being discussed here. When the 3PL provider's shipping cost is far smaller than the distributor's own shipping cost, the distributor's expected profit in 3PL model will be larger than the other one even without considering the set up cost. At the same time, if the 3PL provider's shipping time is far shorter than the distributor's own shipping time, similar situation will happen as the smaller shipping cost one. Actually, in our real life, both of the above two situations are not practical.

Observation IV

- From table 5.4, we find that as the price elasticity becomes larger, the distributor's decreasing ratio starts to decrease. This is because when the product's demand is quite price sensitive, the 3PL provider's service will be more efficient compared to the transportation by the distributor himself.

Table 5.4: The comparison with respect to the price elasticity

	TDI Scheme			3PL Model				
b	π_c^*	π_d^*	π_m^*	π_c^*	π_d^*	π_m^*	π_{3PL}^*	ϖ
2.00	2.17	1.09	0.54	2.76	0.69	0.17	0.34	36.98%
2.25	1.10	0.53	0.29	1.45	0.33	0.10	0.18	36.75%
2.50	0.55	0.26	0.15	0.76	0.16	0.06	0.10	36.31%
2.75	0.29	0.13	0.08	0.41	0.09	0.03	0.05	35.44%
3.00	0.15	0.07	0.05	0.23	0.04	0.02	0.03	34.23%
3.25	0.08	0.04	0.02	0.13	0.02	0.01	0.02	32.77%
3.50	0.05	0.02	0.01	0.07	0.01	0.01	0.01	31.10%
3.75	0.03	0.01	0.01	0.04	0.01	0.00	0.01	29.24%

So the distributor should choose to use 3PL provider for the delivery of the product of which the demand is more price sensitive.

□ **End of chapter.**

Chapter 6

Conclusion and Discussion

Supply chains involving long distance transportation of fresh products have become increasingly common in global as well as domestic markets. The difference of making decisions in a traditional and fresh product decentralized supply chain is that the producer and distributor have to take the quantity decline during the long distance transportation into consideration. The demand model in my study is dependent on the freshness of the product when it reaches the market and the deterioration function $\theta(t)$ and the decay function $m(t)$ take general forms.

6.1 TDI Concluding Remarks

A generalized conclusion of the TDI scheme is that the coordination strategy can be reached when the product is highly perishable and the demand is very sensitive to the distributor's selling price. The producer can make the original decentralized supply chain coordinated by offering the TDI scheme under which the producer and distributor can enhance their own expected profit respectively. At the same time, the whole decentralized supply chain profit is increased to the centralized supply chain profit.

In detail, the TDI scheme I defines the transportation cost into two parts which divided by $\bar{q}$. Our research result implies that under the optimal transportation discount factor α^* and $\bar{q} = q_c^*$, the supply chain get coordinated. In addition, under the above scheme, the producer will not have motivation to let the distributor order more than q_c^*, which will decrease its own profit.

The TDI scheme II makes the whole supply chain coordinated through the product quantity loss, freshness compensation and buy back contracts. The quantity loss and freshness depend on the arrival time t, order quantity q and the buy back contract depends on the selling price. The arrival time may be known by the producer but the selling price may not be obtained since the price decision is made by the distributor. That leads to the TDI scheme III, which the coordination only depends on the arrival time period t and q. The producer will prefer the scheme III since the unknown selling price in scheme II may bring price risk. This definitely may let the producer suffer from the loss of a certain amount of profit. Meanwhile, we still figure it out that the producer's and the distributor's shares of the profit will be $\alpha_1\pi_c$ and $(1-\alpha_1)\pi_c$ for scheme II and $\alpha_2\pi_c$ and $(1-\alpha_2)\pi_c$ for scheme III. To ensure that each party's profit with scheme II and III is not less than the traditional decentralized supply chain, we can find the upper bound and lower bound of α_1 and α_2.

$$\{\alpha_1 \text{ and } \alpha_2\} \in [\pi_m^*/\pi_c, 1-\pi_d^*/\pi_c] \tag{6.1}$$

6.2 3PL Concluding Remarks

For the supply chain business model involving 3PL provider, it is different from the model in paper [4]. Our model considers a supply chain with three parties, the producer, the distributor and the 3PL provider. The distributor will sign a contract with the 3PL provider to help with the shipping issue. Therefore, the transportation cost and the corresponding perishability risks are born by the distributor. Meanwhile, we also let the 3PL provider set a guaranteed arrival time for the product. If the product arrives late, the 3PL should have to pay the penalty cost to the distributor. This let the 3PL provider share portion of the transportation risk for the distributor. The optimal decisions faced by the producer, the distributor and the 3PL provider in the decentralized supply chain are studied respectively in my thesis. Based on the optimal decisions in the fully centralized supply chain as a benchmark, we develop an coordination mechanism so that the decentralized supply chain acts in an coordinated way. Specifically, the coordination mechanism consists of two parts: (i) The wholesale pricing policy between the producer and the distributor. This policy states a time dependent wholesale price discount for every unit sold by the producer. (ii) The penalty factor policy between the distributor and the 3PL provider. This policy shows the time dependent penalty factor offered by the 3PL provider to the distributor. Different from the wholesale pricing policy, the penalty factor will be increasing to the arrival time t. The 3PL provider is willing to pay more penalty cost to the distributor to achieve the cooperation.

6.3 Future Work

The investigation of fresh product supply chain with Third-Party Logistics provider is a relatively new line of research. I have considered the situation where the 3rd party logistics providers provide a guaranteed arrival time to the distributor. In reality, there may be other alternatives among the three parties in the supply chain. For example, the 3rd party logistics provider may offer volume discount to the distributor to motivate the distributor's order behavior. The volume discount can be continuous or discrete. This is an interesting problem for further research. Another possible extension of this paper is to let the producer bear the transportation risk. This is related to the common model in export business "CIF" (Cost Insurance and Freight). Apparently the uncoordinated and coordinated decisions and schemes in CIF transactions will be different. I expect that the framework of the model and the related results I have established in the current thesis can serve as a basis for my further studies.

□ **End of chapter.**

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